SEM-4 (MATHEMATICS GENERAL)
ANUP RAY

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Solution of Home work Problems

1) a) Since $1+(-1)=0 \notin H$, hence we find that H is a not of Subgroup of Z.

b) Clearly, $H \neq \phi$. Let $a=2n$ and $b=2m$ be two elements of H. Then $a-b=2n-2m=2(n-m)$ $\in H$. H is a Subgroup of Z.

2) Let $a=ghg^{-1}$ and $b=gh_1g^{-1}$ be two elements of K.

Then $ab^{-1}=ghg^{-1}(gh_1g^{-1})^{-1}$

$=ghg^{-1}((g^{-1})^{-1}h_1^{-1}g^{-1})$

$=ghg^{-1}gh_1^{-1}g^{-1}$

$=ghh_1^{-1}g^{-1}$ ---- (1)

Now $h, h_1 \in H$ and H is a subgroup of G.
Hence $hh_1^{-1} \in H$. Then from (1) above we get

$ab^{-1}=g(hh_1^{-1})g^{-1} \in gHg^{-1}$

Hence K is a sub-group of G.

Theorem 5: The intersection of any two subgroups of a group $(G,*)$ is again a subgroup of $(G,*)$.

Proof: Let S_1 and S_2 be any two subgroups of $(G,*)$.
We have $S_1 \cap S_2 \neq \phi$, since $e \in S_1$ and $e \in S_2$.
Let $a \in S_1 \cap S_2$ and $b \in S_1 \cap S_2$

Now $a \in S_1 \cap S_2 \Rightarrow a \in S_1$ and $a \in S_2$
$b \in S_1 \cap S_2 \Rightarrow b \in S_1$ and $b \in S_2$

Since S_1 and S_2 form subgroups under the group $(G, *)$, we have

$$a \in S_1,\ b \in S_1 \Rightarrow ab^{-1} \in S_1$$
$$a \in S_2,\ b \in S_2 \Rightarrow ab^{-1} \in S_2$$

Finally, $ab^{-1} \in S_1$, $ab^{-1} \in S_2 \Rightarrow ab^{-1} \in S_1 \cap S_2$

Thus $a \in S_1 \cap S_2$, $b \in S_1 \cap S_2 \Rightarrow ab^{-1} \in S_1 \cap S_2$

Therefore $S_1 \cap S_2$ forms a subgroup of $(G, *)$.

<u>Theorem 6</u>: The union of two subgroups is a subgroup if and only if one is contained in the other.

Proof Left for students as exercise.

<u>Home work</u>: Let $GL(2, R)$ be the group of all non-singular 2×2 matrices over R. Show that each of the following sets is a subgroup of $GL(2, R)$

(i) $H = \left\{ \begin{bmatrix} a & 0 \\ c & d \end{bmatrix} \in GL(2, R) \;\middle|\; ad \neq 0 \right\}$,

(ii) $H = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \in GL(2, R) \;\middle|\; \text{either } a \text{ or } b \neq 0 \right\}$